

一个新的直觉模糊蕴涵算子

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摘 要:在直觉模糊集理论基础上,结合模糊蕴涵的概念,构造了一个新的蕴涵算子,证明了该算子满足边界性、正则性、单调性等一些重要性质,在此基础上,证明了该蕴涵算子和直觉模糊交运算可构成直觉模糊剩余格。

关键词:直觉模糊集;蕴涵算子;剩余格

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直觉模糊集(Intuitionistic fuzzy set, IFS)是 Atanassov 1986 年提出的^[1],是在 Zadeh 经典模糊集的隶属函数基础上增加了非隶属函数,隶属函数和非隶属函数既能描述“亦此亦彼”的模糊概念,又可描述“非此非彼”的中立状态,更加细腻地刻画客观世界的模糊性本质.因此直觉模糊集理论比模糊集更易处理不完备信息,引起众多学者的关注和研究,并在决策、近似推理等领域得到了广泛应用.1986 年 Atanassov 给出了直觉模糊集理论及其运算,1989 年他和 Gargov 又给出了区间值直觉模糊集的概念^[2],并对其进行了研究.在直觉模糊集中,逻辑推理是研究的热点,蕴涵算子在逻辑推理中又起重要作用,所以直觉模糊蕴涵算子的构造研究是重要的研究方向之一.张军政等人提出了一种新的直觉模糊伪 S-蕴涵^[3],并证明了该 S-蕴涵的一些重要性质.薛占熬等人对广义的直觉模糊蕴涵及剩余格进行了研究^[4].Lin 对直觉模糊蕴涵算子的表达式及性质进行了研究^[5].徐泽水对基于直觉模糊蕴涵的直接聚类分析进行了研究^[6].周雷对基于直觉模糊蕴涵算子的直觉模糊粗糙集的刻画进行了研究^[7].Ali 对基于连接函数的模糊蕴涵算子族进行了研究^[8].Renata 对区间值直觉模糊蕴涵的构造进行了研究^[9].这些研究都是对直觉模糊蕴涵算子的构造做了深入研究.正是如此,本文在直觉模糊集理论基础上,构造了一个新的蕴涵算子,并且证明了该蕴涵算子满足边界性、单调性和正则性,同时还证明了该蕴涵算子的其他一系列重要性质.最后证明了该蕴涵算子和直觉模糊交运算可构成直觉模糊剩余格。

1 基础知识

定义 1^[1-2] 设 U 是一个非空论域,则 U 上的一个直觉模糊集 A 定义为 $A(x) = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in U \}$, 其中: $\mu_A(x): U \rightarrow [0, 1]$ 和 $\nu_A(x): U \rightarrow [0, 1]$ 分别代表 A 的隶属函数和非隶属函数,且对于 A 上的所有 $x \in U$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ 成立,当 $\nu_A(x) = 1 - \mu_A(x)$ 时, A 退化为一般的模糊集。

定义 2^[1-2] 设 A, B 是 U 上的两个直觉模糊集, $A(x) = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in U \}$, $B(x) = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in U \}$, 则 A, B 的包含关系、相等关系定义如下:

$A \subseteq B$ 当且仅当 $\mu_A(x) \leq \mu_B(x), \nu_B(x) \leq \nu_A(x)$;

$A = B$ 当且仅当 $\mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x)$ 。

定义 3^[1-2] 设 A, B 是 U 上的两个直觉模糊集, $A(x) = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in U \}$,

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$B(x) = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in U \}$, 则 A, B 的交、并和补运算定义如下:

$$1) A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle \mid x \in U \};$$

$$2) A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle \mid x \in U \};$$

$$3) A^c(x) = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in U \}.$$

定义 4^[7] 直觉模糊集上常数的定义: $(\alpha, \beta) = \{ \langle x, \alpha, \beta \rangle \mid x \in U \}$, 其中, $\alpha, \beta \in [0, 1]$, 且 $\alpha + \beta \leq 1$. 在直觉模糊集中, 全集 $U = (1, 0) = \{ \langle x, 1, 0 \rangle \mid x \in U \}$, 空集 $\phi = (0, 1) = \{ \langle x, 0, 1 \rangle \mid x \in U \}$.

2 直觉模糊蕴涵算子与性质

本节在直觉模糊集理论基础上, 构造出一新的直觉模糊蕴涵算子, 证明该蕴涵算子的一系列重要性质.

定义 5 设 A, B 是 U 上的两个直觉模糊集, $A(x) = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in U \}$,

$B(x) = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in U \}$, 则 A, B 的直觉模糊蕴涵算子 $(\rightarrow_{IFS})$ 定义如下:

$$A \rightarrow_{IFS} B = \{ \langle x, \nu_A(x) \vee \mu_B(x), \mu_A(x) \wedge \nu_B(x) \rangle \mid x \in U \}.$$

其中, $\mu_A(x) \leq 1 - \nu_A(x)$, $\nu_A(x) \leq 1 - \mu_A(x)$, 记: $1 - \mu_A(x) = \neg \mu_A(x)$.

定理 1 在直觉模糊集中, 直觉模糊蕴涵 $\rightarrow_{IFS}$ 满足边界条件

$$(1) \phi \rightarrow_{IFS} \phi = U; (2) \phi \rightarrow_{IFS} U = U; (3) U \rightarrow_{IFS} \phi = \phi; (4) U \rightarrow_{IFS} U = U.$$

证明

$$(1) \phi \rightarrow_{IFS} \phi = \{ \langle x, 1 \vee 0, 0 \wedge 1 \rangle \mid x \in U \} = \{ \langle x, 1, 0 \rangle \mid x \in U \} = U;$$

$$(2) \phi \rightarrow_{IFS} U = \{ \langle x, 1 \vee 1, 0 \wedge 0 \rangle \mid x \in U \} = \{ \langle x, 1, 0 \rangle \mid x \in U \} = U;$$

$$(3) U \rightarrow_{IFS} \phi = \{ \langle x, 0 \vee 0, 1 \wedge 1 \rangle \mid x \in U \} = \{ \langle x, 0, 1 \rangle \mid x \in U \} = \phi;$$

$$(4) U \rightarrow_{IFS} U = \{ \langle x, 0 \vee 1, 1 \wedge 0 \rangle \mid x \in U \} = \{ \langle x, 1, 0 \rangle \mid x \in U \} = U.$$

定理 2 设 A 是任意一个直觉模糊集, 则直觉模糊蕴涵 $\rightarrow_{IFS}$ 满足下面的正则性:

$$(1) A \rightarrow_{IFS} \phi = A^c; (2) \phi \rightarrow_{IFS} A = U;$$

$$(3) U \rightarrow_{IFS} A = A; (4) U \rightarrow_{IFS} U = U.$$

证明

$$(1) A \rightarrow_{IFS} \phi = \{ \langle x, \nu_A(x) \vee 0, \mu_A(x) \wedge 1 \rangle \mid x \in U \} = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in U \} = A^c;$$

$$(2) \phi \rightarrow_{IFS} A = \{ \langle x, 1 \vee \mu_A(x), 0 \wedge \nu_A(x) \rangle \mid x \in U \} = \{ \langle x, 1, 0 \rangle \mid x \in U \} = U;$$

$$(3) U \rightarrow_{IFS} A = \{ \langle x, 0 \vee \mu_A(x), 1 \wedge \nu_A(x) \rangle \mid x \in U \} = A;$$

$$(4) U \rightarrow_{IFS} U = \{ \langle x, 0 \vee 1, 1 \wedge 0 \rangle \mid x \in U \} = \{ \langle x, 1, 0 \rangle \mid x \in U \} = U.$$

定理 3 设 A, B, C 是任意的 3 个直觉模糊集, 则直觉模糊蕴涵 $\rightarrow_{IFS}$ 关于第一个变量单调递减, 关于第二个变量单调递增, (1) 如果 $A \subseteq B$, 则 $B \rightarrow_{IFS} C \subseteq A \rightarrow_{IFS} C$; (2) 如果 $B \subseteq C$, 则 $A \rightarrow_{IFS} B \subseteq A \rightarrow_{IFS} C$.

证明 (1) 因为 $B \rightarrow_{IFS} C = \{ \langle x, \nu_B(x) \vee \mu_C(x), \mu_B(x) \wedge \nu_C(x) \rangle \mid x \in U \}$, $A \rightarrow_{IFS} C = \{ \langle x, \nu_A(x) \vee \mu_C(x), \mu_A(x) \wedge \nu_C(x) \rangle \mid x \in U \}$, 且由 $A \subseteq B$ 知, $\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \nu_B(x)$. 故, $\nu_B(x) \vee \mu_C(x) \leq \nu_A(x) \vee \mu_C(x)$, $\mu_B(x) \wedge \nu_C(x) \geq \mu_A(x) \wedge \nu_C(x)$ 故, $B \rightarrow_{IFS} C \subseteq A \rightarrow_{IFS} C$.

(2) 因为 $A \rightarrow_{IFS} B = \{ \langle x, \nu_A(x) \vee \mu_B(x), \mu_A(x) \wedge \nu_B(x) \rangle \mid x \in U \}$; $A \rightarrow_{IFS} C = \{ \langle x, \nu_A(x) \vee \mu_C(x), \mu_A(x) \wedge \nu_C(x) \rangle \mid x \in U \}$, 且由 $B \subseteq C$ 知, $\mu_B(x) \leq \mu_C(x)$, $\nu_B(x) \geq \nu_C(x)$. 所以, $\nu_A(x) \vee \mu_B(x) \leq \nu_A(x) \vee \mu_C(x)$; $\mu_A(x) \wedge \nu_B(x) \geq \mu_A(x) \wedge \nu_C(x)$, 故, $A \rightarrow_{IFS} B \subseteq A \rightarrow_{IFS} C$.

定理 4 设 A, B, C 是任意的 3 个直觉模糊集, 则直觉模糊蕴涵 $\rightarrow_{IFS}$ 满足如下性质:

$$(1) A \rightarrow_{IFS} (B \rightarrow_{IFS} C) = B \rightarrow_{IFS} (A \rightarrow_{IFS} C);$$

$$(2) A \rightarrow_{IFS} B = (B)^c \rightarrow_{IFS} (A)^c;$$

$$(3) A \rightarrow_{IFS} B = B \rightarrow_{IFS} A = U, \text{ 则 } A = B;$$

$$(4) (A \cup B) \rightarrow_{IFS} C = (A \rightarrow_{IFS} C) \cap (B \rightarrow_{IFS} C);$$

$$(5) (A \cap B) \rightarrow_{IFS} C = (A \rightarrow_{IFS} C) \cup (B \rightarrow_{IFS} C);$$

$$(6) A \rightarrow_{IFS} (B \cap C) = (A \rightarrow_{IFS} B) \cap (A \rightarrow_{IFS} C);$$

- (7) $A \rightarrow_{IFS} (B \cup C) = (A \rightarrow_{IFS} B) \cup (A \rightarrow_{IFS} C)$;
 (8) 若 $A \subseteq B$ 且 $C \subseteq D$, 则 $D \rightarrow_{IFS} A \subseteq C \rightarrow_{IFS} B$;
 (9) $B \subseteq ((A \rightarrow_{IFS} B) \rightarrow_{IFS} B)$;
 (10) $((A \rightarrow_{IFS} C) \cap (B \rightarrow_{IFS} C) \subseteq ((A \cap B) \rightarrow_{IFS} C)$;
 (11) $((A \rightarrow_{IFS} B) \cap (A \rightarrow_{IFS} C)) \subseteq (A \rightarrow_{IFS} (B \cup C))$;
 (12) $A \cap B \subseteq A \cap (A \rightarrow_{IFS} B)$.

证明 (1) 因为 $B \rightarrow_{IFS} C = \{ \langle x, v_B(x) \vee \mu_C(x), \mu_B(x) \wedge v_C(x) \rangle \mid x \in U \}$, $A \rightarrow_{IFS} (B \rightarrow_{IFS} C) = \{ \langle x, v_A(x) \vee v_B(x) \vee \mu_C(x), \mu_A(x) \wedge \mu_B(x) \wedge v_C(x) \rangle \mid x \in U \}$, $A \rightarrow_{IFS} C = \{ \langle x, v_A(x) \vee \mu_C(x), \mu_A(x) \wedge v_C(x) \rangle \mid x \in U \}$, $B \rightarrow_{IFS} (A \rightarrow_{IFS} C) = \{ \langle x, v_B(x) \vee v_A(x) \vee \mu_C(x), \mu_B(x) \wedge \mu_A(x) \wedge v_C(x) \rangle \mid x \in U \}$, 所以, $v_A(x) \vee v_B(x) \vee \mu_C(x) = v_B(x) \vee v_A(x) \vee \mu_C(x)$, $\mu_A(x) \wedge \mu_B(x) \wedge v_C(x) = \mu_B(x) \wedge \mu_A(x) \wedge v_C(x)$. 故, $A \rightarrow_{IFS} (B \rightarrow_{IFS} C) = B \rightarrow_{IFS} (A \rightarrow_{IFS} C)$.

(2) 因为 $A \rightarrow_{IFS} B = \{ \langle x, v_A(x) \vee \mu_B(x), \mu_A(x) \wedge v_B(x) \rangle \mid x \in U \}$, $(B)^c \rightarrow_{IFS} (A)^c = \{ \langle x, \mu_B(x) \vee v_A(x), v_B(x) \wedge \mu_A(x) \rangle \mid x \in U \}$. 所以, $v_A(x) \vee \mu_B(x) = \mu_B(x) \vee v_A(x)$, $\mu_A(x) \wedge v_B(x) = v_B(x) \wedge \mu_A(x)$. 故, $A \rightarrow_{IFS} B = (B)^c \rightarrow_{IFS} (A)^c$.

(3) 因为 $A \rightarrow_{IFS} B = \{ \langle x, v_A(x) \vee \mu_B(x), \mu_A(x) \wedge v_B(x) \rangle \mid x \in U \}$, $B \rightarrow_{IFS} A = \{ \langle x, v_B(x) \vee \mu_A(x), \mu_B(x) \wedge v_A(x) \rangle \mid x \in U \}$, 若 $A \rightarrow_{IFS} B = B \rightarrow_{IFS} A = U$, 则, $v_A(x) \vee \mu_B(x) = v_B(x) \vee \mu_A(x) = 1$, $\mu_A(x) \wedge v_B(x) = \mu_B(x) \wedge v_A(x) = 0$, 知 $v_A(x) = v_B(x)$, $\mu_A(x) = \mu_B(x)$. 故 $A = B$.

(4) 因为 $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), v_A(x) \wedge v_B(x) \rangle \mid x \in U \}$, $(A \cup B) \rightarrow_{IFS} C = \{ \langle x, (v_A(x) \wedge v_B(x)) \vee (\mu_A(x) \vee \mu_B(x)) \wedge v_C(x) \mid x \in U \rangle \}$, $A \rightarrow_{IFS} C = \{ \langle x, v_A(x) \vee \mu_C(x), \mu_A(x) \wedge v_C(x) \mid x \in U \rangle \}$, $B \rightarrow_{IFS} C = \{ \langle x, v_B(x) \vee \mu_C(x), \mu_B(x) \wedge v_C(x) \mid x \in U \rangle \}$, $(A \rightarrow_{IFS} C) \cap (B \rightarrow_{IFS} C) = \{ \langle x, (v_A(x) \wedge v_B(x)) \vee \mu_C(x), (\mu_A(x) \vee \mu_B(x)) \wedge v_C(x) \mid x \in U \rangle \}$ 所以, $(A \cup B) \rightarrow_{IFS} C = (A \rightarrow_{IFS} C) \cap (B \rightarrow_{IFS} C)$.

(5) 因为 $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), v_A(x) \vee v_B(x) \mid x \in U \rangle \}$, $(A \cap B) \rightarrow_{IFS} C = \{ \langle x, v_A(x) \vee v_B(x) \vee \mu_C(x), \mu_A(x) \wedge \mu_B(x) \wedge v_C(x) \mid x \in U \rangle \}$, $A \rightarrow_{IFS} C = \{ \langle x, v_A(x) \vee \mu_C(x), \mu_A(x) \wedge v_C(x) \mid x \in U \rangle \}$, $B \rightarrow_{IFS} C = \{ \langle x, v_B(x) \vee \mu_C(x), \mu_B(x) \wedge v_C(x) \mid x \in U \rangle \}$, $(A \rightarrow_{IFS} C) \cup (B \rightarrow_{IFS} C) = \{ \langle x, v_A(x) \vee v_B(x) \vee \mu_C(x), \mu_A(x) \wedge \mu_B(x) \wedge v_C(x) \mid x \in U \rangle \}$ 所以, $(A \cap B) \rightarrow_{IFS} C = (A \rightarrow_{IFS} C) \cup (B \rightarrow_{IFS} C)$.

(6) 因为 $B \cap C = \{ \langle x, \mu_B(x) \wedge \mu_C(x), v_B(x) \vee v_C(x) \mid x \in U \rangle \}$, $A \rightarrow_{IFS} (B \cap C) = \{ \langle x, v_A(x) \vee (\mu_B(x) \wedge \mu_C(x)), \mu_A(x) \wedge (v_B(x) \vee v_C(x)) \mid x \in U \rangle \}$, $A \rightarrow_{IFS} B = \{ \langle x, v_A(x) \vee \mu_B(x), \mu_A(x) \wedge v_B(x) \mid x \in U \rangle \}$, $A \rightarrow_{IFS} C = \{ \langle x, v_A(x) \vee \mu_C(x), \mu_A(x) \wedge v_C(x) \mid x \in U \rangle \}$, 所以, $(A \rightarrow_{IFS} B) \cap (A \rightarrow_{IFS} C) = \{ \langle x, v_A(x) \vee (\mu_B(x) \wedge \mu_C(x)), \mu_A(x) \wedge (v_B(x) \vee v_C(x)) \mid x \in U \rangle \}$, $(A \rightarrow_{IFS} B) \cap (A \rightarrow_{IFS} C) = \{ \langle x, v_A(x) \vee (\mu_B(x) \wedge \mu_C(x)), \mu_A(x) \wedge (v_B(x) \vee v_C(x)) \mid x \in U \rangle \}$, 故, $A \rightarrow_{IFS} (B \cap C) = (A \rightarrow_{IFS} B) \cap (A \rightarrow_{IFS} C)$.

(7) 因为 $B \cup C = \{ \langle x, \mu_B(x) \vee \mu_C(x), v_B(x) \wedge v_C(x) \mid x \in U \rangle \}$, $A \rightarrow_{IFS} (B \cup C) = \{ \langle x, v_A(x) \vee \mu_B(x) \vee \mu_C(x), \mu_A(x) \wedge v_B(x) \wedge v_C(x) \mid x \in U \rangle \}$, $A \rightarrow_{IFS} B = \{ \langle x, v_A(x) \vee \mu_B(x), \mu_A(x) \wedge v_B(x) \mid x \in U \rangle \}$, $A \rightarrow_{IFS} C = \{ \langle x, v_A(x) \vee \mu_C(x), \mu_A(x) \wedge v_C(x) \mid x \in U \rangle \}$, $(A \rightarrow_{IFS} B) \cup (A \rightarrow_{IFS} C) = \{ \langle x, v_A(x) \vee \mu_B(x) \vee \mu_C(x), \mu_A(x) \wedge v_B(x) \wedge v_C(x) \mid x \in U \rangle \}$, 所以, $A \rightarrow_{IFS} (B \cup C) = (A \rightarrow_{IFS} B) \cup (A \rightarrow_{IFS} C)$.

(8) 若 $A \subseteq B$ 且 $C \subseteq D$, 则 $D \rightarrow_{IFS} A \subseteq C \rightarrow_{IFS} B$, 由 $A \subseteq B$, $C \subseteq D$ 知, $\mu_A(x) \leq \mu_B(x)$, $v_A(x) \geq v_B(x)$, $\mu_C(x) \leq \mu_D(x)$, $v_C(x) \geq v_D(x)$. $D \rightarrow_{IFS} A = \{ \langle x, v_D(x) \vee \mu_A(x), \mu_D(x) \wedge v_A(x) \mid x \in U \rangle \}$, $C \rightarrow_{IFS} B = \{ \langle x, v_C(x) \vee \mu_B(x), \mu_C(x) \wedge v_B(x) \mid x \in U \rangle \}$, $v_D(x) \vee \mu_A(x) \leq v_C(x) \vee \mu_B(x)$, $\mu_D(x) \wedge v_A(x) \geq \mu_C(x) \wedge v_B(x)$, 所以, $D \rightarrow_{IFS} A \subseteq C \rightarrow_{IFS} B$.

(9) 因为 $B \subseteq ((A \rightarrow_{IFS} B) \rightarrow_{IFS} B)$, $A \rightarrow_{IFS} B = \{ \langle x, v_A(x) \vee \mu_B(x), \mu_A(x) \wedge v_B(x) \mid x \in U \rangle \}$, $(A \rightarrow_{IFS} B) \rightarrow_{IFS} B = \{ \langle x, (\mu_A(x) \wedge v_B(x)) \vee \mu_B(x), (v_A(x) \vee \mu_B(x)) \wedge v_B(x) \mid x \in U \rangle \}$, 且 $\mu_B(x) \leq$

$(\mu_A(x) \wedge \nu_B(x)) \vee \mu_B(x), \nu_B(x) \geq (\nu_A(x) \vee \mu_B(x)) \wedge \nu_B(x)$, 故, $B \subseteq ((A \rightarrow_{IFS} B) \rightarrow_{IFS} B)$.

(10) 因为 $A \rightarrow_{IFS} C = \{ \langle x, \nu_A(x) \vee \mu_C(x), \mu_A(x) \wedge \nu_C(x) \rangle \mid x \in U \}$, $B \rightarrow_{IFS} C = \{ \langle x, \nu_B(x) \vee \mu_C(x), \mu_B(x) \wedge \nu_C(x) \rangle \mid x \in U \}$, $(A \rightarrow_{IFS} C) \cap (B \rightarrow_{IFS} C) = \{ \langle x, (\nu_A(x) \wedge \nu_B(x)) \vee \mu_C(x), (\mu_A(x) \vee \mu_B(x)) \wedge \nu_C(x) \rangle \mid x \in U \}$, $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle \mid x \in U \}$, $((A \cap B) \rightarrow_{IFS} C) = \{ \langle x, \nu_A(x) \vee \nu_B(x) \vee \mu_C(x), (\mu_A(x) \vee \mu_B(x)) \wedge \nu_C(x) \rangle \mid x \in U \}$, 由 $(\nu_A(x) \wedge \nu_B(x)) \vee \mu_C(x) \leq \nu_A(x) \vee \nu_B(x) \vee \mu_C(x)$, 得, $((A \rightarrow_{IFS} C) \cap (B \rightarrow_{IFS} C)) \subseteq ((A \cap B) \rightarrow_{IFS} C)$.

(11) 因为 $A \rightarrow_{IFS} B = \{ \langle x, \nu_A(x) \vee \mu_B(x), \mu_A(x) \wedge \nu_B(x) \rangle \mid x \in U \}$, $A \rightarrow_{IFS} C = \{ \langle x, \nu_A(x) \vee \mu_C(x), \mu_A(x) \wedge \nu_C(x) \rangle \mid x \in U \}$, $(A \rightarrow_{IFS} B) \cap (A \rightarrow_{IFS} C) = \{ \langle x, \nu_A(x) \vee (\mu_B(x) \wedge \mu_C(x)), \mu_A(x) \wedge (\nu_B(x) \vee \nu_C(x)) \rangle \mid x \in U \}$, $B \cup C = \{ \langle x, \mu_B(x) \vee \mu_C(x), \nu_B(x) \wedge \nu_C(x) \rangle \mid x \in U \}$, $A \rightarrow_{IFS} (B \cup C) = \{ \langle x, \nu_A(x) \vee \mu_B(x) \vee \mu_C(x), \mu_A(x) \wedge \nu_B(x) \wedge \nu_C(x) \rangle \mid x \in U \}$, 由 $\nu_A(x) \vee (\mu_B(x) \wedge \mu_C(x)) \leq \nu_A(x) \vee \mu_B(x) \vee \mu_C(x)$, $\mu_A(x) \wedge (\nu_B(x) \vee \nu_C(x)) \geq \mu_A(x) \wedge \nu_B(x) \wedge \nu_C(x)$, 得, $((A \rightarrow_{IFS} B) \cap (A \rightarrow_{IFS} C)) \subseteq (A \rightarrow_{IFS} (B \cup C))$.

(12) 因为 $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle \mid x \in U \}$, $A \cap (A \rightarrow_{IFS} B) = \{ \langle x, \mu_A(x) \wedge (\nu_A(x) \vee \mu_B(x)), \nu_A(x) \vee (\mu_A(x) \wedge \nu_B(x)) \rangle \mid x \in U \}$, 由 $\mu_A(x) \wedge \mu_B(x) \geq \mu_A(x) \wedge (\nu_A(x) \vee \mu_B(x))$, $\nu_A(x) \vee \nu_B(x) \leq \nu_A(x) \vee (\mu_A(x) \wedge \nu_B(x))$, 得, $A \cap B \subseteq A \cap (A \rightarrow_{IFS} B)$.

3 直觉模糊剩余格

本节首先介绍伴随对和剩余格的基本概念, 然后在蕴涵算子的基础上证明该蕴涵算子和直觉模糊交运算可构成直觉模糊剩余格.

定义 6^[10] 设 P 是偏序集, P 上的二元运算 $\otimes$ 与 $\rightarrow$ 叫作互为伴随, 若以下条件成立:

(M₀) $\otimes: P \times P \rightarrow P$ 是单调递增的;

(R₀) 关于第一变量是不增的, 关于第二变量是不减的;

(A) $a \otimes b \leq c$ 当且仅当 $a \leq b \rightarrow c$, $a, b, c \in P$. ($\otimes, \rightarrow$) 叫作 P 上的伴随对.

定义 7^[10] 有界格 L 叫作剩余格(residuated lattice), 若

(i) L 上有伴随对($\otimes, \rightarrow$);

(ii) $\langle L, \otimes, 1 \rangle$ 是带单位元 1 的交换半群, 这里 1 是 L 的最大元; 这时 L 常记作 $\langle L, \otimes, \rightarrow \rangle$.

根据定义 6 和定义 7, 给出剩余格的等价定义, 如下.

定义 8^[10] 三元组 $\langle IFS, \cap, \rightarrow \rangle$ 称为一个剩余格, 如果下列条件成立:

(1) $\cap$ 是不减的, 即 $A \subseteq B$ 时, $A \cap C \subseteq B \cap C$;

(2) $\rightarrow$ 关于第二个变量不减, 即 $B \subseteq C$ 时, $A \rightarrow B \subseteq A \rightarrow C$;

(3) $\rightarrow$ 关于第一个变量不增, 即 $A \subseteq B$ 时, $B \rightarrow C \subseteq A \rightarrow C$;

(4) $A \cap B \subseteq C$ 当且仅当 $A \subseteq B \rightarrow C$;

(5) $\cap$ 满足结合律, 即 $(A \cap B) \cap C = (A \cap B \cap C)$;

(6) $\cap$ 满足交换律, 即 $A \cap B = B \cap A$;

(7) $\cap$ 以 U 为单位元, 即 $U \cap A = A$.

定理 5 三元组 $\langle IFS, \cap, \rightarrow_{IFS} \rangle$ 可构成剩余格.

证明 设 A, B, C 是任意的 3 个直觉模糊集, 根据定义 8, 具体证明如下.

(1) 由 $A \subseteq B$ 得 $\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \nu_B(x)$, 所以, $\mu_A(x) \wedge \mu_C(x) \leq \mu_B(x) \wedge \mu_C(x)$, $\nu_A(x) \vee \nu_C(x) \geq \nu_B(x) \vee \nu_C(x)$, 即 $A \cap C \subseteq B \cap C$. 满足定义 8(1).

(2) 和 (3) 由单调性容易得证.

(3) 充分性: 由 $A \cap B \subseteq C$ 得, $\mu_A(x) \wedge \mu_B(x) \leq \mu_C(x)$, $\nu_A(x) \vee \nu_B(x) \geq \nu_C(x)$, $(\mu_A(x) \wedge \mu_B(x)) \vee \neg \mu_B(x) \leq \mu_C(x) \vee \neg \mu_B(x) = \mu_A(x) \vee \neg \mu_B(x) \wedge (\mu_B(x) \vee \neg \mu_B(x)) \leq \mu_C(x) \vee \neg \mu_B(x) = \mu_A(x) \vee \neg \mu_B(x) \leq \mu_C(x) \vee \neg \mu_B(x) = \mu_A(x) \leq \mu_C(x)$, 即, $\mu_A(x) \leq \nu_B(x) \vee \mu_C(x)$, $\nu_A(x) \vee \nu_B(x) \geq \nu_C(x) =$

$(v_A(x) \vee v_B(x)) \wedge \neg v_B(x) \geq v_C(x) \wedge \neg v_B(x), = v_A(x) \wedge \neg v_B(x) \vee (v_B(x) \wedge \neg v_B(x)) \geq v_C(x) \wedge \neg v_B(x) = v_A(x) \wedge \neg v_B(x) \geq v_C(x) \wedge \neg v_B(x) = v_A(x) \geq v_C(x)$, 即, $v_A(x) \geq \mu_B(x) \wedge v_C(x)$, 所以 $A \cap B \subseteq C$, 推得 $A \subseteq B \rightarrow C$. 必要性: 由 $\mu_A(x) \leq v_B(x) \vee \mu_C(x), v_A(x) \geq \mu_B(x) \wedge v_C(x)$, 得, $\mu_A(x) \wedge \neg v_B(x) \leq (v_B(x) \vee \mu_C(x)) \wedge \neg v_B(x) = \mu_A(x) \wedge \neg v_B(x) \leq (v_B(x) \wedge \neg v_B(x)) \vee (\mu_C(x) \wedge \neg v_B(x)) = \mu_A(x) \wedge \neg v_B(x) \leq \mu_C(x) \wedge \neg v_B(x) = \mu_A(x) \leq \mu_C(x)$, 所以, $\mu_A(x) \wedge \mu_B(x) \leq \mu_C(x)$.

$v_A(x) \geq \mu_B(x) \wedge v_C(x) = v_A(x) \vee \neg \mu_B(x) \geq (\mu_B(x) \wedge v_C(x)) \vee \neg \mu_B(x) = v_A(x) \vee \neg \mu_B(x) \geq (\mu_B(x) \vee \neg \mu_B(x)) \wedge (v_C(x) \vee \neg \mu_B(x)) = v_A(x) \vee \neg \mu_B(x) \geq v_C(x) \vee \neg \mu_B(x) = v_A(x) \geq v_C(x)$ 所以, $v_A(x) \vee v_B(x) \geq v_C(x)$.

综上得, $A \cap B \subseteq C$ 当且仅当 $A \subseteq B \rightarrow C$ 成立.

(5)、(6) 易证.

(7) $U \cap A = (1, 0) \cap \{ \langle x, \mu_A(x), v_A(x) \rangle \mid x \in U \} = \{ \langle x, 1 \wedge \mu_A(x), 0 \vee v_A(x) \rangle \mid x \in U \} = A$.

综上所述, 三元组 $\langle I(2^L), \cap, \rightarrow_{IFS} \rangle$ 可构成剩余格.

4 小 结

蕴涵算子是逻辑推理系统中研究的重点,也是近年来研究的热点,本文结合直觉模糊集概念构造出了一新的直觉模糊蕴涵算子,其满足边界性,正则性,单调性,证明了蕴涵的算子的一些重要性质,证明了该蕴涵算子和直觉模糊交运算可构成直觉模糊剩余格.可进一步探究该蕴涵算子的代数性质.

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A New Intuitionistic Fuzzy Implication Operator

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Abstract: A new implication operator is constructed by combining the concept of fuzzy implication based on the theory of intuitionistic fuzzy sets. It has been proved that the operator satisfies some important properties such as boundary, regularity, and monotone. On this foundation, it is proved that this implication operator and intuitionistic fuzzy intersection operation can form the intuitionistic fuzzy residuated lattice.

Keywords: intuitionistic fuzzy sets; implication operator; residuated lattice